Turing Undecidability and the Efficient Market Hypothesis

Abstract: The use of computers to simulate financial markets is commonplace and well understood. This preliminary study examines the reverse relationship: the use of financial markets to simulate computers. Turing undecidability (the computer analogue of Godel's incompleteness theorem) has unexpected implications on the mathematical validity of the Efficient Market Hypothesis (EMH).

Fundamental Problem

Let's say I write a contract:

"I hereby promise to pay the bearer of this note $1000 one year from today."

This is, of course, a bond; and if I sell this on an open market I can expect to get $1000 minus the interest.

But what happens if I write the contract as follows:

"I hereby promise to pay the bearer of this note $500 plus $500 one year from today."

I should expect that this note would sell for the same amount as the previous note. Indeed, the Efficient Market Hypothesis demands it; otherwise, someone could make a pure arbitrage profit by buying the cheaper and selling the dearer.

Of course, I need not restrict myself to addition; I could substitute any mathematical function or computer program. In particular, I can write a contract as follows:

"I promise to pay the bearer of this note the output of Turing machine X one year from today, where Turing machine X is the following: ..."

The Efficient Market Hypothesis guarantees that this contract is priced perfectly and instantaneously, which means that the market has simulated Turing machine X and instantaneously determined its output.

Theoretical computer scientists ought to hear alarm bells ringing now; an efficient market can do something that no Turing machine can do: determine the output of a Turing machine. For example, let's say that the market uses algorithm P to price securities:

market price = P(security)

Then what happens if you write a contract where Turing machine X is:

X = P(self) + $1

That is, Turing machine X runs the pricing algorithm on the security, and adds a dollar to the result. This means that either P gives the wrong market price, or P fails to give an answer.

The logic behind this is similar to the the ancient paradox:

1. The following sentence is true.
2. The previous sentence is false.

Godel Variations

Let's say I have two very large prime numbers, X and Y. I multiply them together to get Z (Z = X Y). Now I write a contract, saying:

"I promise to pay the bearer of this note $1000 if Z is prime, but $0 if Z is not prime."

The strong form of EMH says that no inside knowledge (in this case, the primality of Z) will help the investor to make profits. So, the market has done a factorization of Z instantaneously.

We can generalize the above example. For instance, I can write a contract saying:

I promise to pay the bearer of this note $1000 if Fermat's Last Theorem is true.

Chris Oei

Last updated 11/27/2005 12:04:51 AM